EE 230

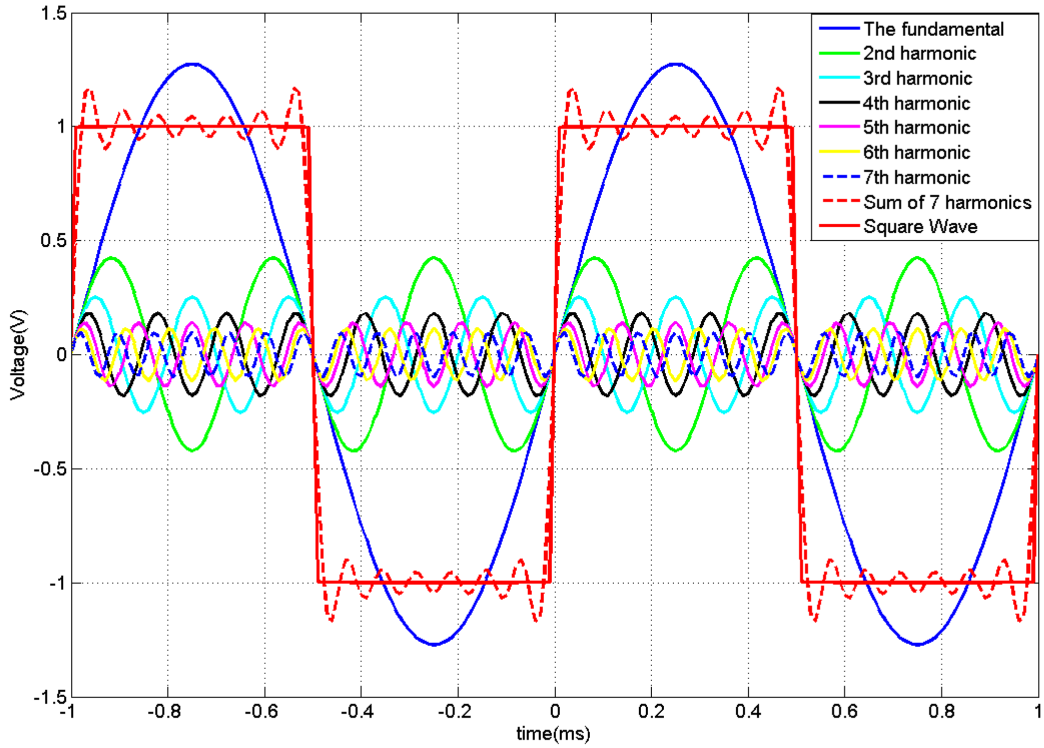
HW2 Solutions

Problem 1

1. A square wave can be represented a sum of an infinite number of sine waves as follows:

$$x_{square} = \left(\frac{4}{\pi}\right)\sum_{k=1}^{\infty}\frac{\sin(2\pi ft(2k-1))}{2k-1}$$

a. When K=1 , we obtain the fundamental
b. When K=2,….,7 , we obtain the 2^{nd},3^{rd}, …..,7^{th} harmonics respectively
c. Sum up the 1^{st} 7 harmonics



Matlab code for problem 1:

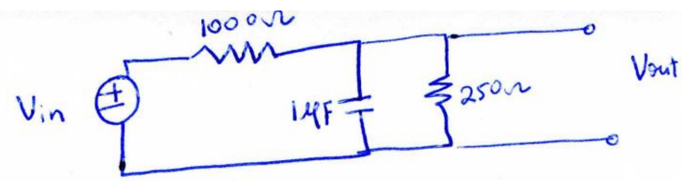
```
clear all
close all

f=1000;                % frequency
t=-1e-3:1e-5:1e-3;     % time

x_out=0;
for i=1:1000
    x = (4/pi)*sin((2*i-1)*2*pi*f.*t)/(2*i-1);  % One harmonic at a time
```

```
    x_out = x_out + x;                          % Add up the harmonics
    if i==1
        plot(t*1e3,x,'b','linewidth',2.5),
        xlabel('time(ms)','fontsize',14),ylabel('Voltage(V)','fontsize',14)
        hold on
    end
    if i==2
        plot(t*1e3,x,'g','linewidth',2.5),
        xlabel('time(ms)','fontsize',14),ylabel('Voltage(V)','fontsize',14)
        hold on
    end
    if i==3
        plot(t*1e3,x,'c','linewidth',2.5),
        xlabel('time(ms)','fontsize',14),ylabel('Voltage(V)','fontsize',14)
        hold on
    end
    if i==4
        plot(t*1e3,x,'k','linewidth',2.5),
        xlabel('time(ms)','fontsize',14),ylabel('Voltage(V)','fontsize',14)
        hold on
    end
    if i==5
        plot(t*1e3,x,'m','linewidth',2.5),
        xlabel('time(ms)','fontsize',14),ylabel('Voltage(V)','fontsize',14)
        hold on
    end
    if i==6
        plot(t*1e3,x,'y','linewidth',2.5),
        xlabel('time(ms)','fontsize',14),ylabel('Voltage(V)','fontsize',14)
        hold on
    end
    if i==7
        plot(t*1e3,x,'b--','linewidth',2.5),
        xlabel('time(ms)','fontsize',14),ylabel('Voltage(V)','fontsize',14)
        hold on
        plot(t*1e3,x_out,'r--','linewidth',2.5),
        hold on
    end
    if i==1000
        plot(t*1e3,x_out,'r','linewidth',2.5),
        xlabel('time(ms)','fontsize',14),ylabel('Voltage(V)','fontsize',14)
        hold on
    end
end
set(get(gcf,'CurrentAxes'),'FontSize',14)
grid on
legend('The fundamental','2nd harmonic','3rd harmonic','4th harmonic','5th
harmonic','6th harmonic','7th harmonic','Sum of 7 harmonics','Square Wave')
```

Problem 2

a. THD = 0.02%
b. Operating Range = 20Hz-40KHz
c. Gain deviation = the gain is flat to within -3dB. That means they are only guaranteeing the gain to be within 3dB over the entire frequency range

Problem 3



a) $Z_1 = R_1$; $Z_2 = R_2$; $Z_3 = \frac{1}{j\omega C}$

$$Z = Z_2 // Z_3 = \frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{R_2/j\omega C}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{1 + j\omega R_2 C}$$

$$\Rightarrow V_{out} = V_{in} \cdot \frac{Z}{Z_1 + Z} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{Z}{Z_1 + Z} = \frac{R_2/(1 + j\omega R_2 C)}{R_1 + R_2/(1 + j\omega R_2 C)}$$

$$\Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{R_2}{(1 + j\omega R_2 C) R_1 + R_2} = \frac{R_2}{(R_1 + R_2) + j\omega R_1 R_2 C}}$$

$$\boxed{T(j\omega) = \frac{V_{out}}{V_{in}} = \frac{250}{1250 + 0.25 j\omega}}$$

b) $$\boxed{T(s) = \frac{250}{1250 + 0.25 s}}$$ $s = j\omega$

c) $V_{in} = 0.5 \sin 1000t \Rightarrow \omega = 1000$ rad/s

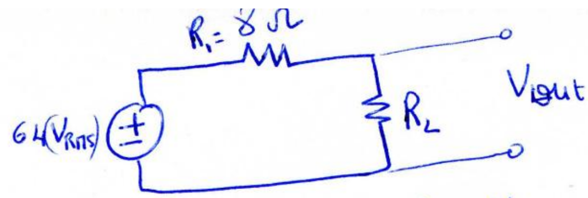
at $\omega = 1000$ rad/s, we have $T(j\omega) = \frac{250}{1250 + j250} = \frac{1}{5 + j} = \frac{5 - j}{(5 + j)(5 - j)} = \frac{5 - j}{25 + 1}$

$$\Rightarrow T(j\omega)\big|_{\omega = 1000} = \frac{5}{26} - \frac{1}{26} j = 0.196 \angle -0.196 \leftarrow \text{rad}$$

$$V_{out}(t) = 0.5 \times 0.196 \times \sin(1000t - 0.196)$$

$$\boxed{V_{out}(t) = 0.098 \sin(1000t - 0.196)}$$

Problem 4



a) * $R_L = 4\,\cancel{8}\,\Omega$ $\Rightarrow V_{out} = \dfrac{4 \times 64}{4+8} = \left(\dfrac{128}{3}\right)V$

Power $= P_L = \dfrac{V_{out}^2}{R_L} = \dfrac{(128/3)^2}{4} = \boxed{113.78\ W}$

* $R_L = 8\Omega \Rightarrow P_L = \dfrac{\left(\frac{8 \times 64}{16}\right)^2}{8} = \boxed{128\ W}$

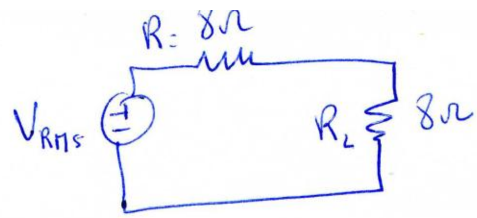
* $R_L = 1000\Omega \Rightarrow P_L = \dfrac{\left(\frac{64 \times 1000}{1008}\right)^2}{1000} = \boxed{4.03\ W}$

b) $P_L = \dfrac{V_{out}^2}{R_L} = \dfrac{\left(\frac{R_L V_{in}}{R_1 + R_L}\right)^2}{R_L}$ V_{in} ?

$\Rightarrow P_L \cdot R_L = \left(\dfrac{R_L}{R_1 + R_L}\right)^2 V_{in}^2$

$\Rightarrow V_{in} = \sqrt{\dfrac{P_L \cdot R_L}{\left(\frac{R_L}{R_1+R_L}\right)}} = \sqrt{\dfrac{128 \times 1000}{\left(\frac{1000}{1008}\right)}} = \boxed{360\ V}$

Problem 5



Assume: 100 W at 8Ω load

$$\Rightarrow P_L = \frac{V_L^2}{R_L} = \frac{\left(\frac{V_{RMS} R_L}{R_L + R}\right)^2}{R_L}$$

$$\Rightarrow 100 = \frac{V_{RMS}^2 \left(\frac{8}{16}\right)^2}{8} \Rightarrow \boxed{V_{RMS} = \sqrt{\frac{100 \times 8}{(0.5)^2}} = 56.56\,V}$$

V_p = Voltage (peak) $= \sqrt{2} \cdot V_{RMS} = \sqrt{2} \times 56.56V = 80V$

$$\boxed{V_p = 80V}$$

Now 100 W at 1000 Ω load

$$\boxed{V_p = V_{RMS} \cdot \sqrt{2} = \sqrt{2} \cdot \sqrt{\frac{100 \times 1000}{\left(\frac{1000}{1008}\right)}} = 448.9\,V}$$

- To achieve the same power at 1000 Ω load, the power amplifier will require ~~at~~ a very high voltage
- AC outlet doesn't provide this voltage
- This high voltage is hazardous.

Problem 6

Problem 6

$$T(s) = 20\,\frac{s+1}{s^2+5s+6}$$

a) $$T(j\omega) = 20\,\frac{j\omega+1}{(j\omega)^2+5j\omega+6} = \frac{20+20j\omega}{(6-\omega^2)+5j\omega}$$

$$|T(j\omega)| = \frac{\sqrt{20^2+20^2\omega^2}}{\sqrt{(6-\omega^2)^2+25\omega^2}} = \frac{20\sqrt{1+\omega^2}}{\sqrt{\omega^4+13\omega^2+36}}$$

b) The maximum gain is 4.38 at $\omega = 2$ rad

c) System is linear $\Longrightarrow$ THD $= 0$

d) $$T_1(j3) = 20\,\frac{j3+1}{15j-3} = 4.13\angle{-0.519}$$

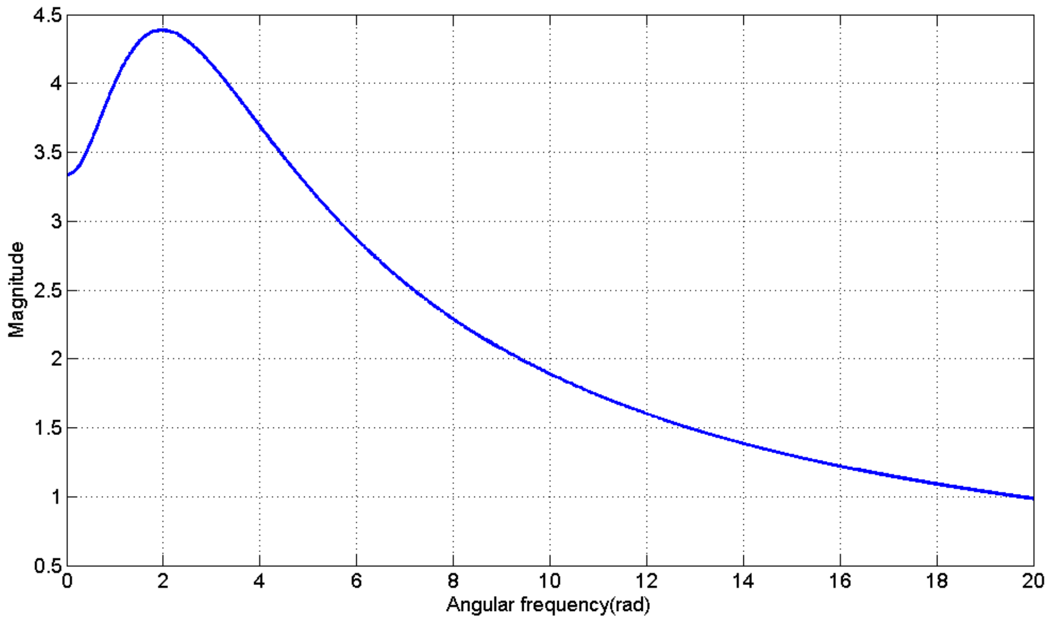
$$T_2(j6) = 20\,\frac{j6+1}{30j-30} = 2.86\angle{-0.951}$$

$$V_{out_1} = 0.4 \times 4.13\ \sin(3t-0.519) = 1.654\sin(3t-0.519)$$

$$V_{out_2} = 0.2 \times 2.86\ \sin(6t-0.951) = 0.57\sin(6t-0.951)$$

$$\boxed{P = \frac{(1.654)^2+(0.57)^2}{2} = 1.532\ W}$$

```
clear all
close all
w=0:0.01:20;

T = (20*sqrt(1+w.^2))./sqrt(w.^4 + 13*w.^2 + 36);
plot(w,T,'linewidth',2.5)
grid on
xlabel('Angular
frequency(rad)','fontsize',14),ylabel('Magnitude','fontsize',14)
set(get(gcf,'CurrentAxes'),'FontSize',14)
```



4.5
4
3.5
3
2.5
2
1.5
1
0.5
Magnitude
0
2
4
6
8
10
12
14
16
18
20
Angular frequency(rad)